

半角模型

$$(1) \triangle AF'E \cong \triangle AFE \checkmark$$

$$(2) BE + DF = EF$$

$$(3) C_{\triangle CEF} = 2AB$$

(4) AE 平分 $\angle BAH$

(5) AF 平分 $\angle DAH$

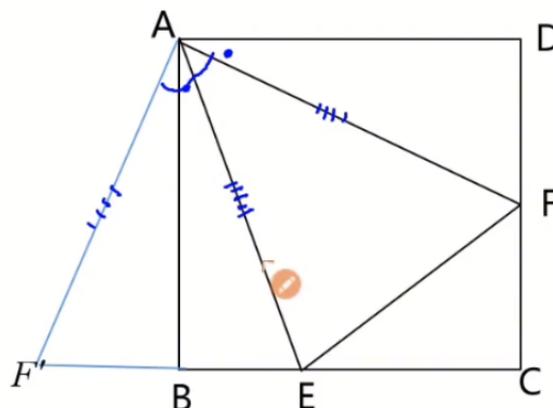
$$(6) \triangle ABE \cong \triangle AHE$$

$$(7) \triangle AHF \cong \triangle ADF$$

$$(8) MA = MF$$

$$(9) NA = NE$$

$$(10) \text{ 若 } \frac{BE}{AB} = \frac{1}{2}, \text{ 则 } \frac{DF}{AD} = \frac{1}{3}$$



将 $\triangle AFD$ 旋转到 ABF' , 因为 $\angle D = 90^\circ$, 所以 F', B, E, C 共线。

因为是旋转得到 $\triangle AF'B$, 所以 $\triangle AF'B \cong \triangle ADF$

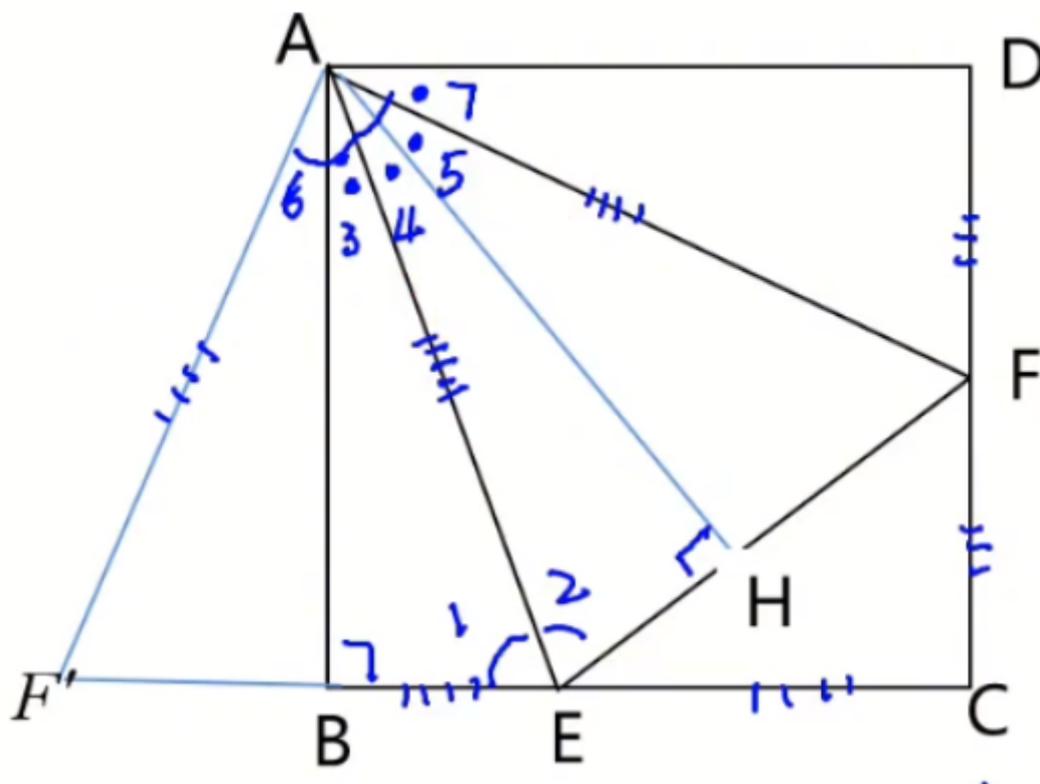
所以 $DF = BF'$, $\angle DAF = \angle BAF'$ 又因为 $\angle BAE + \angle FAD = 45^\circ$ 所以 $\angle F'AB + \angle BAE = \angle F'AE = 45^\circ$

$AF' = AF$, $\angle F'AE = \angle EAF$, $AE = AE$ 根据 ASA , 所以 $\triangle F'AE \cong \triangle EAF$ ①证毕

$BE + DF = BE + BF'$ 旋转得到 $BE + BF' = EF$ ②证毕

$$EF = EF' = BE + BF' = BE + FD$$

$$\therefore C_{\triangle CEF} = CE + CF + EF = CE + CF + BE + FD = BC + CD = 2AB \quad \text{③证毕}$$



因为 $\triangle AF'E \cong \triangle AEF \therefore \angle 1 = \angle 2 \therefore \triangle BAE \cong \triangle EAH$ ⑥证毕

$\therefore \angle 3 = \angle 4$ 因此 AE 是 $\angle BAH$ 平分线, ④证毕

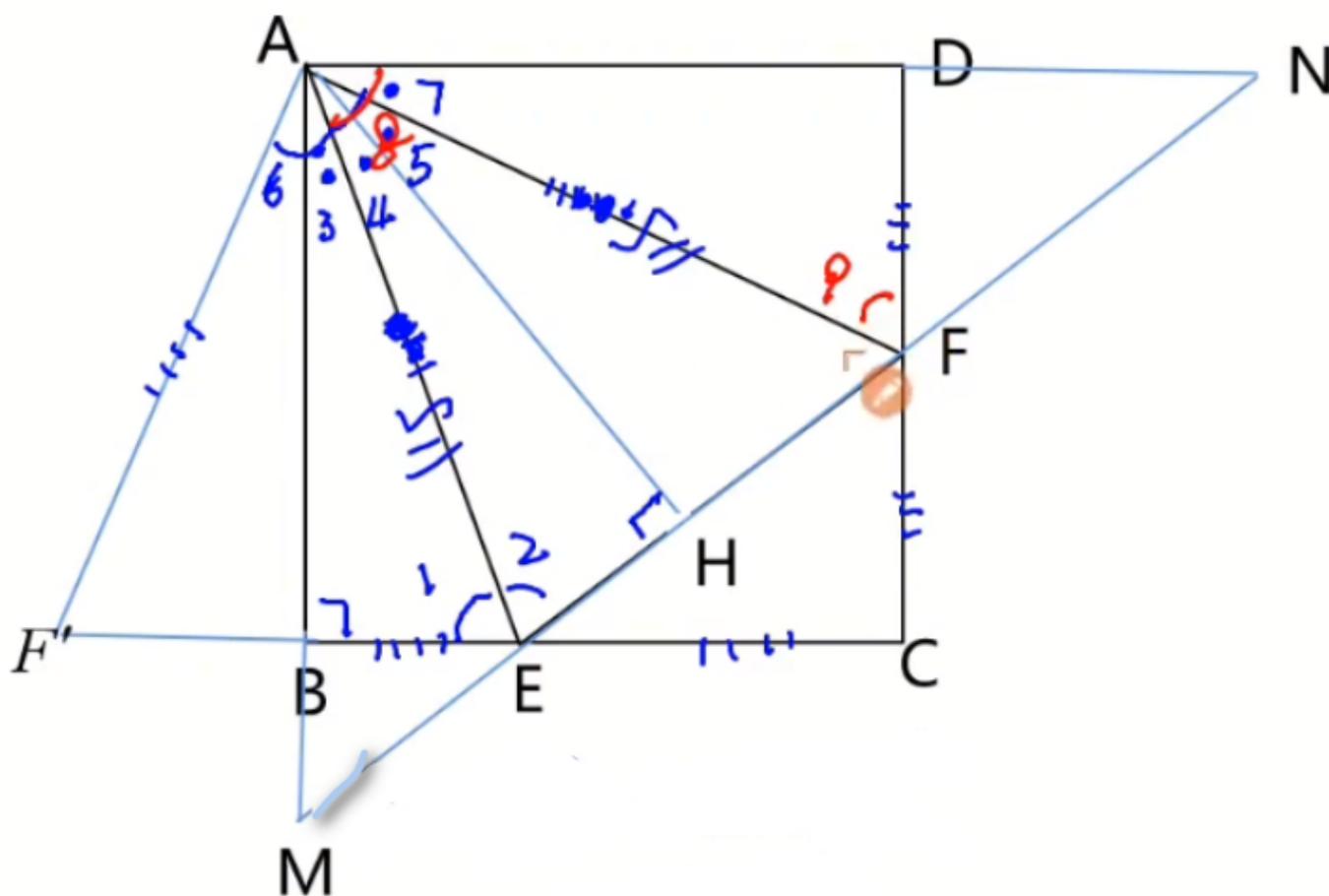
求证: $\angle 5 = \angle 7 \because \angle 4 + \angle 5 = 45^\circ \because \angle 3 = \angle 4 \therefore \angle 3 + \angle 5 = 45^\circ$

$\therefore \angle 7 + \angle 3 = 90^\circ - \angle EAF = 90^\circ - 45^\circ = 45^\circ$

$\therefore \angle 5 = \angle 7$

所以, AF 是 $\angle HAD$ 平分线, ⑤证毕

$\therefore \triangle ADF \cong \triangle AHF$ ⑦证毕



求证: $MA = MF$

因为 $\angle 1 = \angle 2$ 已经证明的全等三角形 $\angle 1 = \angle 8$ 平行边内错角相等 $\therefore \angle 2 = \angle 8$ $\triangle ANE$ 是等腰三角形, $AN = NE$ ⑩ 证毕

$\therefore \triangle AHF \cong \triangle ADF \therefore \angle HFA = \angle 9$

$\therefore \angle 9 = \angle MAF$ 平行线内错角相等

$\therefore \angle HFA = \angle MAF$

所以 $\triangle MAF$ 是一个等腰三角形 $\therefore MF = MA$ ⑩ 证比

⑩ 一看就是 12345 模型了, 因为 $\tan \alpha = \frac{1}{2}, \alpha + \beta = 45^\circ \therefore \tan \beta = \frac{1}{3}$ 证毕, 忘记怎么证明的时候, 回去复习一下 12345 模型即可。